Aspects of Wedge Holography

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based on JHEP 03 (2022) 145 and JHEP 01 (2021) 150

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Outline

1. Background
   - Doubly holographic model
   - Wedge Holography
2. Main Results
   - Spectrum of Wedge holography
   - Effective Action of Wedge holography
   - First Law of entanglement entropy
   - Page curve on codim-2 brane
3. Summary and outlook

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Double holography

Double holography plays an important role in recovering Page curve of Hawking radiations.

- Entanglement entropy (fine-grained entropy) should not exceed black hole entropy (coarse-grained entropy).
- Recover Page curve of eternal BH

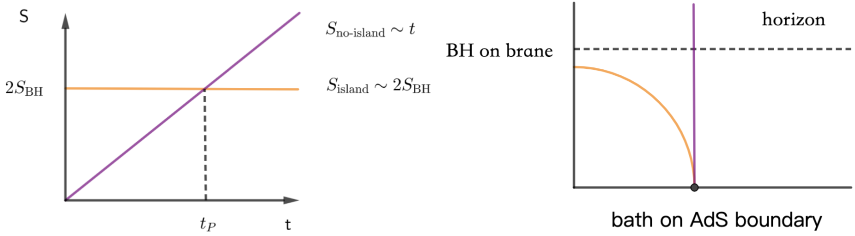

Figure: (left) Page curve of eternal BH; (right) Island in double holography

Outline

Wedge Holography

Classical gravity on wedge W_{d+1} $\simeq$ Quantum gravity on two AdS_d Q
$\simeq$ CFT_{d-1} on Σ

Akal, Kusuki,Takayanagi and Wei, PRD [arXiv:2007.06800]

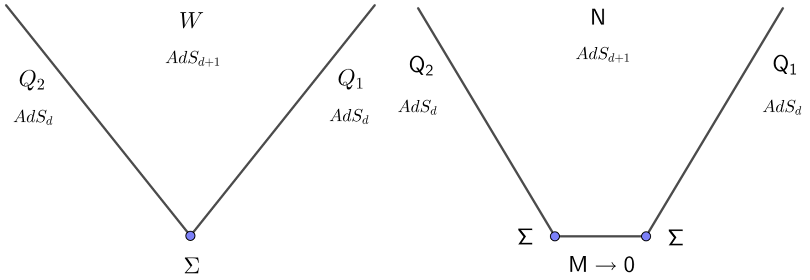

Figure: (left) Geometry of wedge holography; (right) Wedge holography from AdS/BCFT

Proposal of Wedge Holography

- Gravitational action

$$I = \frac{1}{16\pi G_N} \int_N \sqrt{g}(R - 2\Lambda) + \frac{1}{8\pi G_N} \int_{Q_1 \cup Q_2} \sqrt{h}(K - T), \qquad (1)$$

where $T = (d-1)\tanh\rho$ is the brane tension.

- Neumann BC on Q:

$$K^i{}_j - (K - T)h^i{}_j = 0, \qquad (2)$$

where K_{ij} are the extrinsic curvatures.

- Correct Weyl anomaly, entanglement/Rényi entropy, correlation function...

$$\mathcal{A} = \frac{1}{16\pi G_N} \int_\Sigma dx^2 \sqrt{\sigma} \Big(\sinh(\rho) R_\Sigma \Big) \qquad (3)$$

- Correct degree of freedom

Cone holography

Cone holography proposes that a gravity theory in the (d+1)-dimensional cone C is dual to a CFT on (d+1-n)-dimensional defect D.

Miao, PRD [arXiv: 2101.10031]

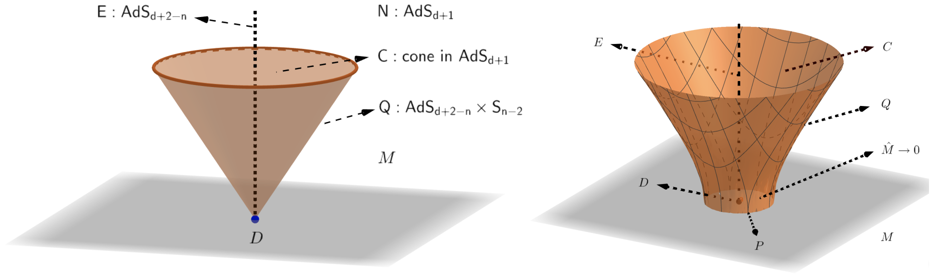

Figure: (left) Geometry of cone holography; (right) Cone holography from AdS/dCFT.

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Perturbation around AdS

- Bulk metric and location of branes

$$ds^2 = dr^2 + \cosh^2(r)\left(\bar{h}^{(0)}_{ij}(y) + \epsilon H(r)\bar{h}^{(1)}_{ij}(y)\right)dy^i dy^j + O(\epsilon^2),$$
$$Q: \ \ r = \pm\rho + O(\epsilon^2), \tag{4}$$

where $\bar{h}^{(0)}_{ij}$ is an AdS metric, $\bar{h}^{(1)}_{ij}$ denotes the perturbation.

- Transverse traceless gauge

$$\bar{D}^i \bar{h}^{(1)}_{ij} = 0, \quad \bar{h}^{(0)ij}\bar{h}^{(1)}_{ij} = 0 \tag{5}$$

- EOM

$$\left(\bar{D}_k\bar{D}^k + 2 - m_g^2\right)\bar{h}^{(1)}_{ij}(y) = 0, \tag{6}$$
$$\cosh^2(r)H''(r) + d\sinh(r)\cosh(r)H'(r) + m_g^2 H(r) = 0, \tag{7}$$

where m_g denotes the mass of gravitons.

Spectrum determined by boundary condition

- Solution

$$H(r)=\text{sech}^{\frac{d}{2}}(r)\left(c_1 P_{\lambda_g}^{\frac{d}{2}}(\tanh r)+c_2 Q_{\lambda_g}^{\frac{d}{2}}(\tanh r)\right), \tag{8}$$

where $\lambda_g=\frac{1}{2}\left(\sqrt{(d-1)^2+4m_g^2}-1\right)$.

- Boundary conditions

$$\text{NBC}: \quad H'(\pm\rho)=0, \tag{9}$$

$$\text{DBC/CBC}: \quad H(\pm\rho)=0, \tag{10}$$

NBC specifies extrinsic curvature, DBC fixes induced metric, CBC specifies the conformal geometry and the trace of extrinsic curvature.

- Constraint of mass

$$\text{NBC}: \ m_g^2\left(P_{\lambda_g}^{\frac{d}{2}-1}(x)Q_{\lambda_g}^{\frac{d}{2}-1}(-x)-P_{\lambda_g}^{\frac{d}{2}-1}(-x)Q_{\lambda_g}^{\frac{d}{2}-1}(x)\right)=0,$$

$$\text{DBC/CBC}: \ P_{\lambda_g}^{\frac{d}{2}}(x)Q_{\lambda_g}^{\frac{d}{2}}(-x)-P_{\lambda_g}^{\frac{d}{2}}(-x)Q_{\lambda_g}^{\frac{d}{2}}(x)=0, \tag{11}$$

where $x=\tanh(\rho)$.

Comments on Spectrum of wedge holography

- Unlike AdS/BCFT and brane world theory, there is a massless mode for wedge holography with NBC.
- The spectrum is non-negative for all kinds of BCs.

$$\begin{cases} m_g^2 \geq 0, & \text{NBC}, \\ m_g^2 > 0, & \text{DBC/CBC}, \end{cases} \tag{12}$$

- In large and small tension limit, except a massless mode, the mass spectrums are the same for all kinds of BCs.

$$\text{NBC/DBC/CBC}: \ m_g^2 \approx \begin{cases} k(k+d-1), & \text{for large } \rho, \\ \frac{k^2\pi^2}{4\rho^2}, & \text{for small } \rho, \end{cases} \tag{13}$$

where k is an integer.

- In small tension limit, the effective theory on the brane is Einstein gravity for NBC.

Outline

Special case I

Perturbative effective action is given by an infinite sum of Pauli-Fierz actions for massive gravity.

- Bulk metric and location of branes

$$ds^2 = dr^2 + \cosh^2(r)\left(\bar{h}^{(0)}_{ij}(y) + \epsilon H(r)\bar{h}^{(1)}_{ij}(y)\right) dy^i dy^j + O(\epsilon^2),$$
$$Q: \ \ r = \pm\rho + O(\epsilon^2). \tag{14}$$

- Orthogonal condition

$$\int_{-\rho}^{\rho} \cosh(r)^{d-2} H^{(m_g)}(r) H^{(m'_g)}(r) dr = \delta^{m_g, m'_g}. \tag{15}$$

- Perturbative effective action (correct sign of kinetic term, ghost-free)

$$I = \sum_{m_g} \int_Q d^d y \sqrt{|\bar{h}^{(0)}|}\Big[-\frac{1}{4}\bar{D}_k \bar{h}^{(m_g)}_{ij} \bar{D}^k \bar{h}^{(m_g)ij} + \frac{1}{2}\bar{D}_k \bar{h}^{(m_g)}_{ij} \bar{D}^i \bar{h}^{(m_g)jk}$$
$$-\frac{d-1}{2}\bar{h}^{(m_g)}_{ij}\bar{h}^{(m_g)ij} - \frac{1}{4} m_g^2 \bar{h}^{(m_g)}_{ij}\bar{h}^{(m_g)ij}\Big] \tag{16}$$

Special case II

In large tension limit, the effective action is composed of CFT effective action and a higher derivative gravity.

- For large tension, the geometry of wedge holography becomes that of AdS/CFT.
- Holographic renormalization in AdS/CFT

$$I_{\text{CFT}} = I_W + I_c \tag{17}$$

- Counterterms on the brane

$$\begin{aligned} I_c \;=\; & \frac{-1}{16\pi G_N}\int_Q \sqrt{|h|}\Big[2(d-1) - 2T + \frac{1}{d-2}\mathcal{R} \\ & + \frac{1}{(d-4)(d-2)^2}\left(\mathcal{R}^{ij}\mathcal{R}_{ij} - \frac{d}{4(d-1)}\mathcal{R}^2\right) + \ldots\Big] \end{aligned} \tag{18}$$

- Effective action of wedge holography

$$I_W = I_{\text{CFT}} - I_c \tag{19}$$

General effective action

For general tension, the effective action is composed of CFT effective action and an infinite tower of higher derivative gravity.

- General effective action

$$I_W = I_{\text{CFT}} - I_c \tag{20}$$

- Support from entanglement entropy [Chen, Myers, Neuenfeld, Reyes, Sandor, JHEP 10 (2020) 166]

$$S_{bulk} = \frac{1}{4G_N}\int dx^{d-1}\sqrt{\sigma_W}, \tag{21}$$

$$S_{brane} = \frac{1}{4G_N}\int dy^{d-2}\sqrt{\sigma_Q}(\frac{1}{d-2} + O(\mathcal{R})) \tag{22}$$

- Support from Weyl anomaly [Hu, Miao, JHEP 03 (2022) 145]

The ghost problem

The effective theory on the brane is derived from Einstein gravity in the bulk, thus it should be ghost-free. However, the effective theory is a higher derivative gravity, which suffers the ghost problem generally.

- Perturbation equation of higher derivative gravity on the brane

$$\Pi_{n=0}^{\infty}(\Box + \frac{2}{L_{\text{eff}}^2} - m_n^2)\delta h_{ij} = 0, \tag{23}$$

- Propagator for $L_{\text{eff}} \to \infty$

$$\begin{aligned} D(p) &\sim \sum_{i=0}^{\infty} \left(\Pi_{j\neq i} \frac{1}{m_j^2 - m_i^2}\right) \frac{1}{p^2 + m_i^2} \\ &\sim \left(\frac{a_0^2}{p^2} - \frac{a_1^2}{p^2 + m_1^2} + \frac{a_2^2}{p^2 + m_2^2} - \frac{a_3^2}{p^2 + m_3^2} + ...\right) \end{aligned} \tag{24}$$

- Eq.(24) implies half of the massive modes are ghost (wrong sign).

Mechanism to eliminate ghost

We argue that the higher derivative gravity on the brane is equivalent to a ghost-free multi-gravity.

- Ghost-free multi-gravity

$$S_N = \lim_{N\to\infty} \sum_{n=1}^{N} \int dy^d \sqrt{|h_n|} \left(\mathcal{R}_n + \frac{m_N^2}{2} \mathcal{L}\left(h_n, h_{n+1}\right) \right) \tag{25}$$

where $\mathcal{R}_n$ is the Ricci scalar of the nth metric h_n and $\mathcal{L}\left(h_n, h_{n+1}\right)$ is the ghost-free interaction between neighboring metrics.

- Eliminating $h_2 ... h_{N-1}$, we get same $\mathcal{R}^2$ terms as HD gravity on brane

$$\begin{aligned} S_N = & \int dy^d \sqrt{|h_1|} \left(\Lambda_1 + c_R \mathcal{R}_1 - \frac{c_{RR}}{m_N^2} \left(\mathcal{R}_1^{\ ij} \mathcal{R}_{1\ ij} - \frac{d}{4(d-1)} \mathcal{R}_1^2 \right) \right) \\ & + \int dy^d \sqrt{|h_N|} \left(\Lambda_N + c_R \mathcal{R}_N - \frac{c_{RR}}{m_N^2} \left(\mathcal{R}_N^{\ ij} \mathcal{R}_{N\ ij} - \frac{d}{4(d-1)} \mathcal{R}_N^2 \right) \right) \\ & + \int dy^d \mathcal{L}_{int}(h_1, h_N) + O(\mathcal{R}^3/m_N^4), \end{aligned} \tag{26}$$

More on ghost-free multi-gravity I

The ghost-free multi-gravity can be derived from Einstein gravity in the bulk. Thus it must be equivalent to the effective higher derivative gravity on the brane.

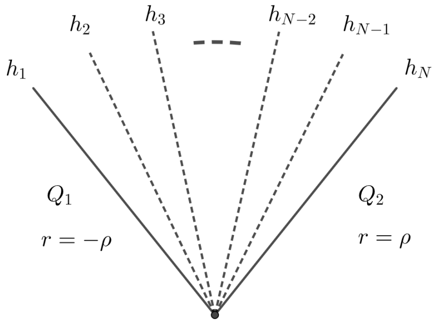


Figure: Deconstruction: make discretization of Einstein gravity and replace the extra dimension r by a series of sites r_b $(1 \leq b \leq N)$. In this way, one can derive ghost-free multi-gravity from the Einstein gravity in the bulk.

More on ghost-free multi-gravity II

The ghost-free multi-gravity can be derived from Einstein gravity.

- Einstein gravity in the bulk

$$I = \int_{-\rho}^{\rho} dr \int dy^d \sqrt{|h|} \left(\mathcal{R}[h] + K^2 - K_{ij}K^{ij} - 2\Lambda \right), \tag{27}$$

- Extrinsic curvature

$$K^i_j[h_n, h_{n+1}] = -m_N \left(\delta^i_j - \left(\sqrt{h_n^{-1} h_{n+1}} \right)^i_j \right), \tag{28}$$

$m_N = N/(2\rho) \backsimeq \partial_r$ denotes derivative.

- Multi-gravity from bulk Einstein gravity

$$I = \lim_{N\to\infty} \sum_{n=1}^{N} \int dy^d \sqrt{|h_n|} \left(\mathcal{R}_n + \frac{m_N^2}{2} \mathcal{L}\left(h_n, h_{n+1}\right) \right) \tag{29}$$

- NBC fix K_1 and K_N while DBC fix h_1 and h_N.

Outline

Review of first law of entanglement entropy

- Relative entropy measures the distance between two states

$$S(\rho_1|\rho_0) = \text{tr}(\rho_1 \ln \rho_1) - \text{tr}(\rho_1 \ln \rho_0). \tag{30}$$

- It can be re-expressed as

$$S(\rho_1|\rho_0) = \Delta\langle H\rangle - \Delta S, \tag{31}$$

$$\Delta\langle H\rangle = \text{tr}(\rho_1 H) - \text{tr}(\rho_0 H), \quad \Delta S = S(\rho_1) - S(\rho_0), \tag{32}$$

where H is the modular Hamiltonian and S is entanglement entropy.

- Positivity of the relative entropy requires

$$\Delta\langle H\rangle \geq \Delta S. \tag{33}$$

- At first-order perturbation, we get first law of entanglement entropy

$$\delta\langle H\rangle = \delta S. \tag{34}$$

- Holographic entanglement entropy

$$S_A = \text{Min}_{\gamma_A}\left[\text{Min}_{\Gamma_A} \frac{A(\Gamma_A)}{4G_N}\right] \quad (35)$$

where $\partial\gamma_A = \partial A$ and $\partial\Gamma_A = \partial\gamma_A$.

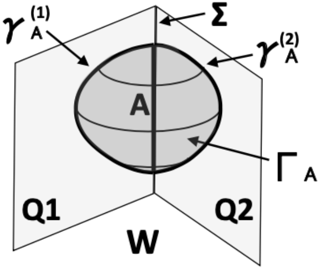


Figure: Vary γ_A on the brane Q to minimize the area of Γ_A in the bulk W.

Holographic entanglement entropy (HEE) of disk

- Bulk metric

$$ds^2 = dr^2 + \cosh^2(r)\frac{dz^2 - dt^2 + dR^2 + R^2 d\Omega_{d-3}^2}{z^2} \tag{36}$$

- HEE of a disk: $R^2 \leq R_0^2$

$$S = \frac{\text{Area}(\Gamma)}{4G_N}, \tag{37}$$

- RT surface Γ:

$$z^2 + R^2 = R_0^2, \;\; t = \text{constant}. \tag{38}$$

First-order Perturbation of HEE

- Perturbative bulk metric

$$ds^2 = dr^2 + \cosh^2(r)\Big(ds^2_{AdS} + \epsilon \sum_{m_g} H^{(m_g)}(r)\bar{h}^{(m_g)}_{ab}(y)dy^a dy^b\Big). \quad (39)$$

- As a minimal surface, RT surface is invariant under first-order perturbation

$$z^2 + r_0^2 = R_0^2,\ \ t = \text{constant}. \quad (40)$$

- First-order perturbation of HEE

$$\delta S = \sum_{m_g} \frac{\epsilon}{8G_N}\int_{-\rho}^{\rho} dr \cosh^{d-2}(r) H^{(m_g)}(r) \int_{R\leq R_0} d^{d-2}y f^{(m_g)}(y), \quad (41)$$

where the expression of $f^{(m_g)}(y)$ is unimportant.

Massive perturbation

- Orthogonal condition

$$\int_{-\rho}^{\rho} \cosh(r)^{d-2} H^{(m_g)}(r) H^{(0)}(r) dr = \delta^{m_g,0}, \tag{42}$$

where $H^{(0)}(r) = 1$ for massless mode.

- Massive mode is irrelevant to first-order perturbation of HEE

$$\delta S = \sum_{m_g} \frac{\epsilon}{8G_N} \int_{-\rho}^{\rho} dr \cosh^{d-2}(r) H^{(m_g)}(r) \int_{R\leq R_0} d^{d-2}y f^{(m_g)}(y) = 0.$$

- Modular Hamiltonian

$$\delta\langle H\rangle = \frac{\pi}{R_0} \int_{R\leq R_0} d^{d-2}x \, z^2 \delta\langle T_{00}\rangle = 0. \tag{43}$$

where T_{00} is the stress tensor which is irrelevant to massive modes.

- First law for Massive perturbation

$$\delta S = \delta\langle H\rangle = 0, \quad \text{for } m_g^2 > 0. \tag{44}$$

Massless perturbation

For massless perturbation, we can also verify the first law of entanglement entropy for wedge holography.

- Effective theory of massless mode is Einstein gravity

$$I_{\rm W} = \frac{1}{16\pi G_N}\int_{-\rho}^{\rho}\cosh^{d-2}(r)dr\int_Q dy^d\sqrt{|\bar{h}|}\Big(R_{\bar{h}}+(d-1)(d-2)\Big).$$

- For Einstein gravity, the first law of entanglement entropy is obeyed

$$\delta S=\delta\langle H\rangle, \quad \text{for } m_g^2=0. \tag{45}$$

- At higher-order perturbations, we have

$$\delta\langle H\rangle \geq \delta S, \quad \text{for } m_g^2=0. \tag{46}$$

- Turn the logic around, derive Einstein equations in the $(d+1)$ bulk from the first law on the $(d-1)$ dimensional corner ?

Outline

Geometry of AdS/dCFT

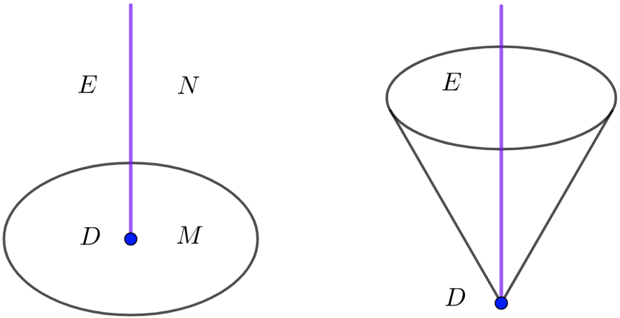


Figure: (left) Geometry of AdS/dCFT (a limit of cone holography); (right) Geometry of cone holography. E is a codim-2 brane in the bulk N, D is a codim-2 defect on the AdS boundary M.

- Gravity can be located on the codim-2 brane E.
- We set black hole on the codim-2 brane E and bath on the AdS boundary M.

Page curve in AdS/dCFT

- bulk metric: codim-2 brane at $r = 0$, AdS bdy at $r \to \infty$

$$ds^2 = dr^2 + \sinh(r)^2 d\theta^2 + \cosh(r)^2 ds^2_{\text{BH}}. \qquad (47)$$

- Page curve

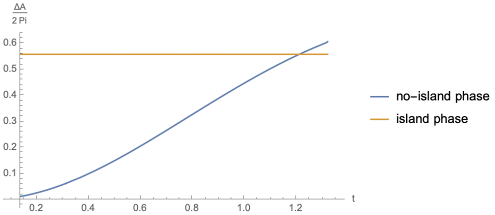


Figure: Page curve on codim-2 brane in $AdS_4/dCFT_3$.

- The extremal surface passing through the horizon (no-island phase) cannot be defined after some finite time.
- This unusual situation does not affect Page curve since it happens after Page time.

Summary and outlook

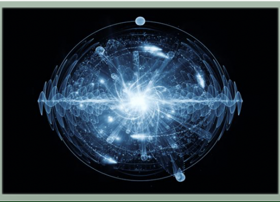
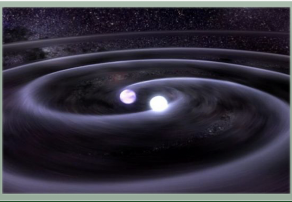
Summary:

- There is massless graviton on the brane in wedge/cone holography with NBC, which is quite different from brane world theory and AdS/BCFT.
- The spectrum is non-negative $m_g^2 \geq 0$ for all kinds of boundary conditions.
- The effective theory on the brane is a ghost-free higher derivative gravity or an equivalent multi-gravity.
- The first law of entanglement entropy is obeyed by wedge holography.
- On codim-2 brane, the no-island phase cannot be defined after some finite time. This unusual situation does not affect Page curve since it happens after Page time.

Outlook:

- Study island and the Page curve of Hawking radiations.
- Derive Einstein equations in bulk from first law on corner of wedge?
- Application of wedge/cone holography in AdS/CMT?

Thanks!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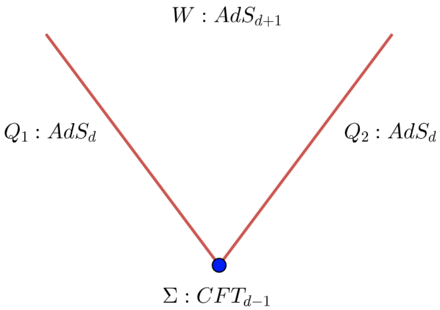

1 classical gravity in bulk W

2 ghost–free higher derivative gravity (massive gravity) on brane Q

3 first law of EE for CFT$_{d-1}$